Modelling and Validation of Quarter Vehicle Traction Model

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ABSTRACT

The present work was undertaken to study mainly the modelling and validation of the quarter vehicle model in order to investigate the dynamic vehicle behaviours of Antilock-Braking System (ABS). The simulation model was developed in MATLAB Simulink software and verified with the experimental data made by Fauzi et al. [1]. The vehicle dynamics behaviours consist of body velocity, wheel longitudinal velocity, and the longitudinal tire slip. By comparing the simulation and the referred data, it can be said that the trend had satisfied each other and can be used as a plan in the ABS control development.

Keywords: Modelling, Validation, Quarter vehicle model.

INTRODUCTION

Today Antilock-Braking System (ABS) was very important in a braking system and played a major role in enhancing vehicle safety. The ABS is an electronic system that helps secure and controls the abrupt stopping of a vehicle. It is a critical safety system that requires accuracy in all equipment, even though on the simulation stage, including brake, vehicle model as well as the control strategy. Fail to do so, and it may cause the developed system could not function properly and thus reduce the braking effectiveness.

In the simulation of ABS, the most important is the credibility of the models. The models should represent the actual system correctly so that the control strategy that was developed can be deemed to be effective. On that basis, a quarter vehicle traction model was developed in this and verified experimentally using data made by Fauzi et al. [1]. The behaviours to be validated were wheel linear velocity, body velocity, longitudinal slip and the stopping distance.

MATHEMATICAL MODELLING

The deriving of mathematical modelling for the 2 DOF vehicle model in Figure 1 is based on the previous approach presented by Sivaramakrishnan [2] and Aly et al. [3]. In this study, some assumptions are made in deriving the dynamics mathematical model of the ABS system. The model only consists of the vehicle longitudinal dynamics, which meant that the vertical and lateral motions could be neglected. Beside that, during braking the
The force of rolling resistance is minimal and can be neglected also. Since this is a quarter-vehicle model, no interaction between the four wheels is taken into consideration. Other assumptions are:

The vehicle model consists of a single body (sprung mass) connected to a wheel (unsprung mass). The sprung mass is assumed to move in the longitudinal direction with the steady velocity of $v$ and the interaction with the road is represented using wheel traction model. Aside from that, the vehicle always remains grounded and the tire never lost contact with the ground during maneuvering. In simulation the drag force of the vehicle during acceleration and braking are neglected. Beside that the rolling resistance force is disregarded since it is very insignificant throughout braking.

![Figure 1. Quarter Car Model](image)

The motion equation of the simplified vehicle according to Newton’s second law can be expressed as:

$$F_x = -m \dot{V}_x$$  \hspace{1cm} (1)

where $m$ is the total mass of the vehicle body and wheel, $\dot{V}_x$ is represented the acceleration of the vehicle and $F_x$ is the tire friction force.

According to Coulomb Law, the tire friction force, $F_x$ also can be defined as:

$$F_x = \mu F_z$$  \hspace{1cm} (2)

Thus

$$-m \dot{V}_x = \mu F_z$$  \hspace{1cm} (3)

where $\mu$ is the road adhesion coefficient and $F_z$ is the total normal force of the vehicle body and wheel and can be defined as:

$$F_z = \mu mg$$  \hspace{1cm} (4)

where $g$ is the gravity mass.

Therefore, the equation (3) can be simplified as:
\[-m\dot{V}_x = \mu mg \quad (5)\]

Thus

\[\dot{V}_x = -\mu g \quad (6)\]

During deceleration, the braking torque is applied to the wheels and causes the speed of wheel and vehicle going to decrease. At the same time, ignored the wheel rolling resistance force because it is too smaller compared to the friction force between the wheel and road. Therefore, according to Newton’s second law, the wheel motion equation can be written as:

\[\tau_\omega - \tau_B = J_\omega \dot{\omega} \quad (7)\]

Where \( \tau_\omega \) is wheel torque, \( \tau_B \) is brake torque, while \( J_\omega \) and \( \dot{\omega} \) are wheel inertia and angular acceleration, respectively.

The wheel torque \( \tau_\omega \) also can be defined as:

\[\tau_\omega = F_x R_t \quad (8)\]

Where \( R_t \) is known as wheel radius.

Therefore,

\[F_x R_t - \tau_B = J_\omega \dot{\omega} \quad (9)\]

By substituting Equation (1) and (5) into (9), equation (10) can be obtained as:

\[\mu mg R_t - \tau_B = J_\omega \dot{\omega} \quad (10)\]

Thus,

\[\dot{\omega} = \frac{\mu mg R_t - \tau_B}{J_\omega} \quad (11)\]

During the vehicle travel at a certain speed, the wheel rotational velocity is corresponding to the vehicle velocity in the forward direction. Brake force is generated at the interface between the wheel and the road surface when the brake is applied and causes the wheel speed to decline. As the force on the wheel’s rises, slippage will occur between the wheel and the road surface and causes the wheel velocity to be lesser than the vehicle velocity [4]. The difference value in velocity during braking is called wheel slip, \( K \) and defined as:

\[K = \frac{V_x - V_w \max(V_x/V_w)}{V_x} \quad (12)\]

The zero value of wheel slip indicates that the wheel and vehicle have a same velocity, while the ratio of one means that the tire is not rotating and the wheels are skidding on the road surface, i.e., the vehicle is beyond controlled. When the wheel slip approaches
at +1, the condition shows the wheel is ‘spinning’ with the vehicle speed at zero value while the wheel is ‘locked’ at zero speed after the wheel slip approaches at -1. The $\mu$ value is obtained using the lookup table based on the experiment on normal road surface. The response from the variation of friction coefficient with slip ratio is used as the input for quarter vehicle ABS model.

**VALIDATION RESULT**

The test was conducted in three conditions of vehicle speed, such as 40 km/h, 60 km/h and 110 km/h. The results from simulation model and then been validated by the experimental data [1] of each speed, respectively. The results of the output variables obtained for each speed respectively are shown through figures 2 to 4. Table 1 shows the tire parameters used in the simulation model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Quarter Vehicle Mass</td>
<td>240 kg</td>
</tr>
<tr>
<td>Rolling Radius</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Wheel Moment Inertia</td>
<td>1.4 kgm$^2$</td>
</tr>
<tr>
<td>Gravity Acceleration</td>
<td>9.81 m/s$^2$</td>
</tr>
</tbody>
</table>

As in the figures, the dotted line represents the responses of the simulation model while the solid line refers to the experimental data. From the observations the simulation data were faster than the experimental results, which resulting error about 1 second the time response. Even that so, the trend of the simulation results was satisfying each other even though there is a small deviation in between the data. These findings may be a direct consequence of difficulty by the driver to maintain a constant ideal speed during maneuvering and the abnormality of the road surface which is totally disregarded in the simulation nature. According to [5] the most important feature in control-oriented model is the trend of the model response. Since the trend of the model responses is almost like the measured responses with a satisfactory level of deviations and errors, the results can be admitted. Apart from that, Rykiel [6] had stated that the reasonable level of deviation between measured and simulated responses should below than 5% of inequalities. According to Sergant, [7], 5% disparities are the maximum allowable error to express the credibility of a simulation. Therefore, based on these statements, it can be concluded that the model is realistic.
Figure 2. Validation results of the sudden braking test at 40 km/h constant speed; (a) Speed Comparison; (b) Wheel Slip and (c) Stopping Distance.
Figure 3. Validation results of the sudden braking test at 60 km/h constant speed; (a) Speed Comparison; (b) Wheel Slip and (c) Stopping Distance.
Figure 4. Validation results of the sudden braking test at 110 km/h constant speed; (a) Speed Comparison; (b) Wheel Speed; (c) Wheel Slip and (d) Stopping Distance.
CONCLUSION

Through this study, a quarter vehicle traction model has been developed and validated in three different speeds, such as 40 km/h, 60 km/h and 110 km/h then been compared with the experimental data at same speeds. The trends between simulation and experiment data shows similarity with satisfactory errors. The deviations in the response, maybe come from simplified modelling assumptions, vehicle speed contribution and surface of road disturbance. The proposed approach reduces the errors considerably by fine tuning both vehicle and tire parameters. Due to the similarities trend between simulations and experimental data of this study indicate that the model could be used for further research to study ABS system performance in real vehicle.

ACKNOWLEDGMENTS

The authors gratefully appreciate the financial support from Majlis Amanah Rakyat (MARA) and Faculty of Mechanical Engineering, Universiti Teknikal Malaysia Melaka (UTeM) for the use of the facilities. Some special thanks go to those who contributed to this study directly or indirectly.

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